

Physical and Chemical Kinetics
Final Examination
07-11-2013

Please use the provided paper sheets to write down the solutions of the problems. Write your name and student ID number on a first page and enumerate all subsequent pages. Do not forget to hand in your paperwork after the examination.

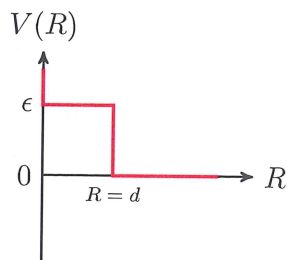
Problem 1

Two interacting hydrogen atoms have laboratory velocities with (x, y, z) components of $(1 \times 10^3 \text{ m/s}, 0, 0)$ and $(0, 1 \times 10^3 \text{ m/s}, 0)$.

- a. Find velocities of the atoms in the center-of-mass frame of reference.
- b. Find the kinetic energy of relative movement of the hydrogen atoms.
- c. Find the total angular momentum of hydrogen atoms relative to the origin of the center-of-mass frame of reference when distance between atoms $\mathbf{r} = (1 \text{ cm}, 1 \text{ cm}, 0)$.

Problem 2

Interaction of particles is described by a repulsive square-well potential with height ϵ at a distance d between particles (see fig.). The relative velocity of the particles before collision is v , impact parameter is b and their relative mass is μ .



- a. What is magnitude of the relative velocity after collision?
- b. Sketch the trajectory of the particle moving in a field with this potential.
- c. Find the distance of the closest approach between the particles as a function of v , b , μ , ϵ , and d .

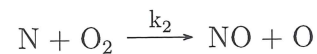
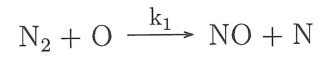
Problem 3

Consider nitrogen at temperature $T = 500$ K. Molecular weight of nitrogen is 28 g/mol.

- Find the most probable magnitude of the velocity of nitrogen molecules at this temperature.
- What is larger - the most probable velocity or the average velocity? Explain why these velocities are not the same.
- Find the sound velocity of nitrogen gas at this temperature.

Problem 4

Formation of the nitrogen oxide proceeds through the Zeldovich mechanism:



- Assuming that rate coefficient k_2 is much larger than k_1 , derive the expression for the rate of formation of the nitric oxide.
- Dissociation of oxygen molecules is sufficient fast that it is possible to assume that this reaction is in equilibrium with the equilibrium constant K_d . Express the rate of formation of NO through the concentration of oxygen and nitrogen molecules.
- Find the mole fraction of NO molecules at exit of a combustion chamber. The combustion gases can be treated as air at temperature 2000 K and pressure 10 atm $\approx 1 \times 10^6$ Pa. The residence time of combustion products in the chamber is ~ 1 ms. The rate coefficient $k_1 = 1.8 \times 10^8 e^{-38370/T}$ m³/mols and the equilibrium constant $k_d = 3.97 \times 10^5 T^{-1/2} [\text{O}_2]^{1/2} e^{-31090/T}$ mol/m³, where T is in K. Molecular weight of air is 29 g/mole.

Physical constants

Electron mass	$m_e = 9.109\,39 \times 10^{-28}$ g
Proton mass	$m_p = 1.672\,62 \times 10^{-24}$ g
Atomic unit of mass	$m_a = \frac{1}{12}m(^{12}\text{C}) = 1.660\,54 \times 10^{-24}$ g
Planck constant	$h = 6.626\,19 \times 10^{-27}$ erg cm $\hbar = 1.054\,57 \times 10^{-27}$ erg cm
Light velocity	$c = 2.997\,92 \times 10^{10}$ cm/s
Electron charge	$e = 1.602\,19 \times 10^{-19}$ C
Avogadro number	$N_A = 6.022\,14 \times 10^{23}$ mol ⁻¹
Molar volume	$V_m = 22.414$ l/mol
Universal gas constant	$R = 8.314 \times 10^7$ erg/mol K
Boltzmann constant	$k = R/N_A = 1.3807 \times 10^{-16}$ erg/K

1 Molecular Motion and Collisions

$$m_1 + m_2 = m'_1 + m'_2 \quad (1.1)$$

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2 \quad (1.2)$$

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \quad (1.3)$$

$$m_1[\mathbf{r}_1 \times \mathbf{v}_1] + m_2[\mathbf{r}_2 \times \mathbf{v}_2] = m_1[\mathbf{r}'_1 \times \mathbf{v}'_1] + m_2[\mathbf{r}'_2 \times \mathbf{v}'_2] \quad (1.4)$$

$$m_1 \frac{d^2\mathbf{r}_1}{dt^2} = \mathbf{F}_{21} \text{ and } m_2 \frac{d^2\mathbf{r}_2}{dt^2} = \mathbf{F}_{12} \quad (1.5)$$

$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 \quad (1.6)$$

$$\mathbf{R} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

$$\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 \quad (1.7)$$

$$\mathbf{V} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$$

$$(m_1 + m_2) \frac{d^2\mathbf{R}}{dt^2} = 0 \quad (1.8)$$

$$\mu \frac{d^2\mathbf{r}}{dt^2} = \mathbf{F}$$

$$\mu = \frac{m_1m_2}{m_1 + m_2} \quad (1.9)$$

$$\mathbf{v}_1 = \mathbf{V} - \frac{m_2}{m_1 + m_2} \mathbf{v} \quad (1.10)$$

$$\mathbf{v}_2 = \mathbf{V} + \frac{m_1}{m_1 + m_2} \mathbf{v}$$

$$\mu[\mathbf{r} \times \mathbf{v}] = \mu[\mathbf{r}' \times \mathbf{v}'] \quad (1.11)$$

$$\frac{\mu v^2}{2} + V(r) = \text{const} \quad (1.12)$$

$$\mathbf{L} = \mu \mathbf{r} \times \mathbf{v} = \text{const} \quad (1.13)$$

$$b = d \cos \frac{\chi}{2}, \text{ when } b \leq d \quad (1.14)$$

$$V(d) = E \left(1 - \frac{b^2}{d^2} \right) \quad (1.15)$$

$$E = \frac{1}{2} \mu v^2 = \frac{1}{2} \mu \dot{r}^2 + \frac{1}{2} \mu r^2 \dot{\theta}^2 + V(r) \quad (1.16)$$

$$\mu v b = \mu r^2 \dot{\theta} \quad (1.17)$$

$$\frac{d\theta}{dr} = \frac{b}{r^2 \sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{\frac{1}{2}\mu}}} \quad (1.18)$$

$$1 - \frac{b^2}{r_0^2} = \frac{V(r_0)}{E} \quad (1.19)$$

$$\chi = \pi - 2b \int_{r_0}^{\infty} \frac{dr}{r^2 \sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}} \quad (1.20)$$

$$b = \frac{A}{2E} \cot \left(\frac{\chi}{2} \right) \quad (1.21)$$

$$dN_{Ar}(v, \alpha, \beta) = \sigma(v, \alpha, \beta) v n_1(v_1) n_2(v_2) \tau d\Omega' \quad (1.22)$$

$$dN_{Ar}(v, \chi, \phi) = \sigma(v, \chi) v n_1(v_1) n_2(v_2) \sin \chi d\chi d\phi \quad (1.23)$$

$$\sigma(v, \alpha, \beta) \sin \alpha d\alpha d\beta = \sigma(v, \chi) \sin \chi d\chi d\phi \quad (1.24)$$

$$\sigma(v) = \int_{\alpha=0}^{\pi} \int_{\beta=0}^{2\pi} \sigma(v, \alpha, \beta) \sin \alpha d\alpha d\beta \quad (1.25)$$

$$\sigma(v, \chi) = \frac{b}{\sin \chi} \left| \frac{db}{d\chi} \right| \quad (1.26)$$

$$\sigma(v, \chi) = \frac{d^2}{2} \cos \frac{\chi}{2} \sin \frac{\chi}{2} \frac{1}{\sin \chi} = \frac{d^2}{4} \quad (1.27)$$

$$\sigma(v, \chi) = \left(\frac{A}{4E} \right)^2 \frac{1}{2 \sin^4 \left(\frac{\chi}{2} \right)} \quad (1.28)$$

$$\sigma(\theta, E = \frac{\hbar^2 k^2}{2\mu}) = |f(\theta, E)|^2 \quad (1.29)$$

2 The Kinetic Theory of Gases

$$N = \int_{v_x=-\infty}^{v_x=\infty} \int_{v_y=-\infty}^{v_y=\infty} \int_{v_z=-\infty}^{v_z=\infty} \iiint_V f(\mathbf{r}, \mathbf{v}, t) dv_x dv_y dv_z dx dy dz \quad (2.1)$$

$$f(\mathbf{r}, \mathbf{v}, t) = \frac{f(\mathbf{r}, \mathbf{v}, t)}{N} \quad (2.2)$$

$$1 = \int_{v_x=-\infty}^{v_x=\infty} \int_{v_y=-\infty}^{v_y=\infty} \int_{v_z=-\infty}^{v_z=\infty} \iiint_V f(\mathbf{r}, \mathbf{v}, t) dv_x dv_y dv_z dx dy dz \quad (2.3)$$

$$f(\mathbf{v}, t) = \iiint_V f(\mathbf{r}, \mathbf{v}, t) dx dy dz \quad (2.4)$$

$$f(\mathbf{v}, t) = \iiint_V f(\mathbf{r}, \mathbf{v}, t) dx dy dz \quad (2.5)$$

$$\begin{aligned} v_x &= v \sin \theta \cos \phi \\ v_y &= v \sin \theta \sin \phi \\ v_z &= v \cos \theta \end{aligned} \quad (2.6)$$

$$d\mathbf{v} = dv_x dv_y dv_z = v^2 dv \sin \theta d\theta d\phi \quad (2.7)$$

$$f(\mathbf{v}, t) d\mathbf{v} = f(v, \theta, \phi, t) v^2 \sin \theta d\theta d\phi dv \quad (2.8)$$

$$f(v, t) = v^2 f(\mathbf{v}, t) \int_{\theta=0}^{\theta=\pi} \int_{\phi=0}^{\phi=2\pi} \sin \theta d\theta d\phi = 4\pi v^2 f(\mathbf{v}, t) \quad (2.9)$$

$$N = \sum_{j=0}^{g-1} f(j, t) \quad (2.10)$$

$$1 = \sum_{j=0}^{g-1} f(j, t) \quad (2.11)$$

$$\tau Z_{12}(\mathbf{v}_1, \mathbf{v}_2) = 2\pi \int_{\chi=0}^{\pi} v \sigma(v, \chi) n_1 n_2 f_1(\mathbf{v}_1, t) f_2(\mathbf{v}_2, t) \tau d\mathbf{v}_1 d\mathbf{v}_2 \sin \chi d\chi \quad (2.12)$$

$$Z_{12} = 2\pi n_1 n_2 \int_{\mathbf{v}_1} \int_{\mathbf{v}_2} \int_{\chi=0}^{\pi} v \sigma(v, \chi) f(\mathbf{v}_1, t) f(\mathbf{v}_2, t) d\mathbf{v}_1 d\mathbf{v}_2 \sin \chi d\chi \quad (2.13)$$

$$\frac{\partial(nf)}{\partial t} + \mathbf{v} \cdot \nabla(nf) + \frac{\mathbf{F}}{m} \cdot \nabla_v(nf) = S_{col} \quad (2.14)$$

$$\begin{aligned} d\mathbf{v}'_1 d\mathbf{v}'_2 &= d\mathbf{V}' d\mathbf{v}' \\ d\mathbf{v}_1 d\mathbf{v}_2 &= d\mathbf{V} d\mathbf{v} \end{aligned} \quad (2.15)$$

$$\begin{aligned} \mathbf{v}'_1 &= \mathbf{v}_1 + \Delta \mathbf{v}_1(v, \chi) \\ \mathbf{v}'_2 &= \mathbf{v}_2 + \Delta \mathbf{v}_2(v, \chi), \end{aligned} \quad (2.16)$$

$$S_{col} = 2\pi n^2 \int_{\chi} \int_{\mathbf{v}_2} v \sigma(v, \chi) \quad (2.17)$$

$$\times \{f(\mathbf{v}_1 + \Delta\mathbf{v}_1, t)f(\mathbf{v}_2 + \Delta\mathbf{v}_2, t) - f(\mathbf{v}_1, t)f(\mathbf{v}_2, t)\} d\mathbf{v}_2 \sin \chi d\chi$$

$$f(\mathbf{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} \quad (2.18)$$

$$f(\mathbf{v}) = f(v_x)f(v_y)f(v_z), \quad (2.19)$$

$$f(v_x)dv_x = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{mv_x^2}{2kT}} dv_x \quad (2.20)$$

$$\langle \mathbf{v} \rangle = \int_{\mathbf{v}} \mathbf{v} f(\mathbf{v}) d\mathbf{v} \quad (2.21)$$

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} \quad (2.22)$$

$$\langle v \rangle = \int_{v=0}^{\infty} v f(v) dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_{v=0}^{\infty} v^3 e^{-\frac{mv^2}{2kT}} dv = \left(\frac{8kT}{\pi m}\right)^{1/2} \quad (2.23)$$

$$\left\langle \frac{1}{2}mv^2 \right\rangle = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^{\infty} \left(\frac{1}{2}mv^2\right) v^2 e^{-\frac{mv^2}{2kT}} dv = \frac{3}{2}kT \quad (2.24)$$

$$Z_{12} = \pi d^2 n_1 n_2 \left(\frac{8kT}{\pi \mu_{12}}\right)^{1/2} = \pi d^2 n_1 n_2 \langle v_{12} \rangle \quad (2.25)$$

$$Z = \frac{\pi d^2}{\sqrt{2}} n^2 \langle v \rangle \quad (2.26)$$

$$l = \frac{\langle v \rangle t}{\pi d^2 \langle v_{12} \rangle t} = \frac{1}{\sqrt{2}\pi d^2 n} \quad (2.27)$$

$$\Gamma_n(\mathbf{r}, t) = \int_{\mathbf{v}} v \cos \theta f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} = \int_{\mathbf{v}} \mathbf{v} \cdot \mathbf{k} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \quad (2.28)$$

$$\Gamma_n = \int_{\mathbf{v}} \mathbf{v} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \quad (2.29)$$

$$\Gamma_m = m\Gamma_n \quad (2.30)$$

$$\Gamma_E(\mathbf{r}, t) = \frac{m}{2} \int_{\mathbf{v}} v^3 \cos \theta f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} = \frac{m}{2} \int_{\mathbf{v}} v^2 \mathbf{v} \cdot \mathbf{k} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \quad (2.31)$$

$$\Gamma_E = \frac{m}{2} \int_{\mathbf{v}} v^2 \mathbf{v} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \quad (2.32)$$

$$\Gamma_{mv} = m \int_{\mathbf{v}} \mathbf{v} \mathbf{v} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \quad (2.33)$$

$$\Gamma_n = \frac{1}{4} n \langle v \rangle \quad (2.34)$$

$$\Gamma_\psi = -\frac{\langle v \rangle}{3} l \left(\frac{d \langle \psi n \rangle}{dz} \right)_{z=0} \quad (2.35)$$

$$\begin{aligned} D &= \frac{1}{3} \langle v \rangle l \\ \eta &= \frac{1}{3} n m \langle v \rangle l \\ \lambda &= \frac{1}{3} n C_v \langle v \rangle l \end{aligned} \quad (2.36)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \langle \mathbf{v} \rangle) = R_{col} \quad (2.37)$$

$$\frac{\partial \rho \langle \mathbf{v} \rangle_\alpha}{\partial t} + \frac{\partial \Gamma_{\alpha\beta}}{\partial x_\beta} + n F_\alpha = 0 \quad (2.38)$$

$$\frac{\partial \left(n \left\langle \frac{mv^2}{2} \right\rangle \right)}{\partial t} + \nabla \cdot \Gamma_E + n \mathbf{F} \cdot \mathbf{v} = 0 \quad (2.39)$$

$$\left(\frac{\partial \rho}{\partial t} \right)_x + \left[\frac{\partial(\rho u)}{\partial x} \right]_t = 0 \quad (2.40)$$

$$\left(\frac{\partial p}{\partial x} \right)_t + \rho \left(\frac{\partial u}{\partial t} \right)_x + \rho u \left(\frac{\partial u}{\partial x} \right)_t = 0 \quad (2.41)$$

$$c^2 = \frac{\omega^2}{k^2} = \left(\frac{\partial p}{\partial \rho} \right)_T + \frac{M p_0}{C_V \rho_0^2} \left(\frac{\partial p}{\partial T} \right)_\rho \quad (2.42)$$

$$c^2 = \frac{p_0}{\rho_0} \left(1 + \frac{R}{C_V} \right) = \frac{p_0}{\rho_0} \gamma, \quad \left(\gamma = \frac{C_p}{C_V} \right) \quad (2.43)$$

$$c^2 = \frac{\omega^2}{k^2} = c_0^2 + \frac{4}{3} \frac{i \eta \omega}{\rho_0} \quad (2.44)$$

3 The Kinetic Theory of Dense Phases

$$P(n_1, N) = \frac{N!}{n_1!(N-n_1)!} p^{n_1} q^{N-n_1} \quad (3.1)$$

$$P(n_1; N) = \frac{1}{\sqrt{2\pi N p q}} e^{-\frac{(n_1 - n_1^*)^2}{2N p q}} \quad (3.2)$$

$$\langle (\Delta n_1)^2 \rangle = N p q \quad (3.3)$$

$$P(n_1; N) = \frac{1}{\sqrt{2\pi \langle (\Delta n_1)^2 \rangle}} e^{-\frac{(n_1 - n_1^*)^2}{2 \langle (\Delta n_1)^2 \rangle}} = \frac{1}{\sqrt{2\pi N p q}} e^{-\frac{(n_1 - n_1^*)^2}{2N p q}} \quad (3.4)$$

$$\langle (\Delta x)^2 \rangle = 2Dt \quad (3.5)$$

$$m \frac{dv}{dt} = -\zeta v + f(t) \quad (3.6)$$

$$m \frac{d\langle v \rangle}{dt} = -\zeta \langle v \rangle \quad (3.7)$$

$$\langle x^2 \rangle = \frac{2kT}{\zeta} \left[t - \frac{m}{\zeta} \left(1 - e^{-\zeta t/m} \right) \right] \quad (3.8)$$

$$\langle x^2 \rangle = \frac{kT}{m} t^2 \quad (3.9)$$

$$\langle x^2 \rangle = \frac{2kT}{\zeta} t \quad (3.10)$$

$$D = \frac{kT}{\zeta} \quad (3.11)$$

$$D = \frac{kT}{6\pi\eta a} \quad (3.12)$$

$$\frac{\langle K \rangle}{D} = \frac{q}{kT} \quad (3.13)$$

$$D = \int_0^\infty d\sigma \langle v(0)v(\sigma) \rangle \quad (3.14)$$

$$\langle v(0)v(\sigma) \rangle = \langle v(0)^2 \rangle e^{-\zeta\sigma/m} = \frac{kT}{m} e^{-\zeta\sigma/m} \quad (3.15)$$

4 Chemical Kinetics

$$\sum \nu_i X_i = 0 \quad (4.1)$$

$$\dot{\xi} = \frac{1}{\nu_i} \frac{dn_i}{dt} \quad (4.2)$$

$$-\frac{d[A]}{dt} = k[A]^\alpha [B]^\beta [C]^\gamma \dots, \quad (4.3)$$

$$-\frac{d[A]}{dt} = k[A] \quad (4.4)$$

$$-\frac{d[A]}{dt} = k[A]^2 \quad (4.5)$$

$$-\frac{d[A]}{dt} = k[A][B] \quad (4.6)$$

$$\frac{[A]}{[B]} = \frac{[A]_0}{[B]_0} e^{(b[A]_0 - a[B]_0)kt} \quad (4.7)$$

$$\sigma_R(E) = \pi p d^2 \left(1 - \frac{\Delta E_f^*}{E} \right) \quad (4.8)$$

$$\left(\frac{dn_C}{dt} \right)_f = n_A n_B \int_{\alpha, \beta} \int_{\mathbf{v}_A} \int_{\mathbf{v}_B} \sigma_R(v, \alpha, \beta) v f_A(\mathbf{v}_A) f_B(\mathbf{v}_B) d\mathbf{v}_A d\mathbf{v}_B d\Omega' \quad (4.9)$$

$$\left(\frac{dn_C}{dt}\right)_f = k_f n_A n_B \quad (4.10)$$

$$k_f = \int_{\mathbf{v}_A} \int_{\mathbf{v}_B} \sigma_R(v) v f_A(\mathbf{v}_A) f_B(\mathbf{v}_B) d\mathbf{v}_A d\mathbf{v}_B \quad (4.11)$$

$$k_f = \pi p d^2 \left(\frac{8kT}{\pi\mu_{AB}}\right)^{1/2} e^{-\Delta E_f^*/kT} \quad (4.12)$$

$$k = A e^{-E_a/RT} \quad (4.13)$$

$$k = AT^n e^{-E_a/RT} \quad (4.14)$$

$$k_f = \kappa \frac{kT}{h} \left(K_V^\ddagger\right)' \quad (4.15)$$

$$\left(K_V^\ddagger\right)' = N_A \frac{q_1[(AB)^\ddagger]}{q_1(A)q_2(B)} e^{-\Delta(E_0^\ddagger)/RT} \quad (4.16)$$

$$\frac{k_f}{k_r} = \frac{[C]_{eq}[D]_{eq}}{[A]_{eq}[B]_{eq}} = K_V \quad (4.17)$$

$$\Delta G^{0\ddagger} = \Delta H^{0\ddagger} - T\Delta S^{0\ddagger} = RT \ln K^\ddagger \quad (4.18)$$

$$E_a = \Delta H^{0\ddagger} + RT \quad (4.19)$$

$$k_f = \kappa \frac{kT}{h} N_A \frac{q_1(AB)^\ddagger}{q_1(A)q_1(B)} e^{-\Delta E_0^{0\ddagger}/RT} \quad (4.20)$$

$$\Delta E_0^{0\ddagger}(m) = E_a - mRT \quad (4.21)$$

$$-\frac{d[A]}{dt} = k[A][B][C] \quad (4.22)$$

$$\frac{d[B]}{dt} = -\frac{d[A]}{dt} = \frac{k_1[A][M]}{k_2[M] + k_3} \quad (4.23)$$

$$[\text{NO}_2^*]_s = \frac{k_1 I_0 [\text{NO}_2]}{k_F + (k_{Q1} + k_2)[\text{NO}_2] + k_{Q2}[\text{Xe}]} \quad (4.24)$$

$$\Phi_F = \frac{I_F}{k_1 I_0 [\text{NO}_2]} = \frac{k_F}{k_F + (k_{Q1} + k_2)[\text{NO}_2] + k_{Q2}[\text{Xe}]} \quad (4.25)$$

$$\Phi(\text{O}_2) = \frac{d[\text{O}_2]/dt}{k_1 I_0 [\text{NO}_2]} = \frac{k_2 [\text{NO}_2^*]_s [\text{NO}_2]}{k_1 I_0 [\text{NO}_2]} = \frac{k_2 [\text{NO}_2]}{k_f + (k_{Q1} + k_2)[\text{NO}_2] + k_{Q2}[\text{Xe}]} \quad (4.26)$$

